SABBATICAL LEAVE PROPOSAL

Curtis Paul

Department of Mathematics

Proposed Leave Time: Fall 2017

Full Time Service: Since August 2004, no previous sabbatical

BACKGROUND

For students in science, technology, engineering, and mathematics (STEM), a major shift occurs in mathematical education after completing the calculus series. Until that point, mathematics courses are mostly focused on computation: solving an equation, finding coordinates, computing an integral, and so forth. Afterwards, the focus is much more on the structure of mathematics and understanding why things are true, why computations are done the way they are, why particular structures apply to particular problems. The student must transition from simply doing computations to offering well-constructed arguments about why certain statements are true. Unfortunately, the previous computational experience ill prepares the student for this transition. From the point of view of institutions receiving transfer students from community colleges, STEM students are transferring with insufficient mathematical maturity and insufficient ability to write mathematics and prove theorems. This is part of a long term trend in the weakening of foundational mathematical skills in students entering college. At one time, calculus students were expected to have a strong enough background to start working on these skills, but the main course now for introducing the student to this transition is linear algebra. The course is a combination of computation and proof and is generally taken by STEM majors after Calculus II or Calculus III. It is the culmination of the mathematics courses at the community college level and is required for all Engineering, Physics, Computer Science, and Mathematics majors. Unfortunately, as a three unit course which exposes the community college student for the first time to the concept of proof while covering many computational processes, there is limited time to develop the craft of writing proofs. There are also limited resources. Some classic textbooks, such as Halmos's Finite-Dimensional Vector Spaces, presume enough mathematical maturity to focus almost exclusively on theorems, structure, and proof.

But most modern textbooks focus on computation and easing the student into proof. Where Halmos introduces the definition of a vector space in the first few pages, current textbooks introduce it around week 10 of a semester. There are a few books specifically on the technique of proof, such as <u>Mathematical Proofs: A Transition to Advanced Mathematics</u>, and <u>The Art of Proof: Basic Training for Deeper Mathematics</u>, but these books are both more general and more in depth than is reasonable for us to require that our students use. So, how to address the problem?

Four years ago, Geoffrey Buhl of CSUCI spoke to the mathematics department here at Moorpark about this and voiced these exact concerns. He is currently the leader of the math group under the Aligning Learning Across Schools (ALAS) grant, a five year, multimillion dollar grant shared by the three Ventura County Community Colleges, CSUCI, and Santa Barbara City College whose purpose is to strengthen 2-year to 4-year curricular pathways and promote transfer student success by aligning high-impact curricular practices, systems, and policies across institutions.

PROPOSAL

One of the programs promoted by ALAS is the Virtual Teaching & Learning Collective (V-TLC), which focuses on shared resources such as study guides, study skills aids, and open educational materials, which can benefit students and instructors, leading to affordable learning solutions, at all of the campuses under the grant. I propose developing a guidebook, geared to the community college student taking linear algebra, which introduces and develops the skill of writing proofs specifically in the setting of the course. The structure of a proof is layered. What are the ideas involved? What do the terms mean? What theorems are relevant? How do they fit together? Before writing a proof down, can you explain the sequence of ideas aloud? Students often write sentences that are wrong in statement, flawed in reasoning, and grammatically incorrect.

The guide will provide examples of proofs at varying levels of correctness and quality, ranging from the incorrect, discursive, and obtuse to correct, concise, and clear. It will discuss the ideas involved and how to map them into a quality proof. It will give proofs requiring short answer completions. It will give flowcharts leading the student to proper structure. And it will focus on developing clear mathematical style.

Each of the major topics in the course will be used as examples and the core concepts discussed again and again to solidify the student's grasp and facility. Each of the major types of proof (proof by induction, proof by contradiction, and direct proof) will be discussed at length. When is each appropriate? What are the relative strengths? Which is going to be most appropriate to a particular problem and why?

This will be done by topic, starting with the most basic skills, such as understanding matrix algebra and the relationships between matrix spaces, and moving to more advanced skills and abstract definitions. The organization has to be fluid enough to conform to the demands of the course for different schools and different textbooks, but comprehensive enough to aid in each of those.

The guide will be approximately 100 pages and made available to students and staff by the ALAS committee. I or a member of the ALAS group will be able to maintain the document as part of V-TLC and incorporate comments and additions by other faculty members as part of our normal service hours. Collecting proofs at various levels of correctness, analyzing them, and presenting them to students with discussions of the thought processes involved is too long a task to be covered under our existing service hours without substantially curtailing other service activities for multiple semesters.

BENEFITS

In discussions about this project with colleagues who teach linear algebra, I've gotten very positive responses about the value it would have for their classes. The math group from ALAS has also been very positive about its contribution.

Our district's stated mission is to "become the leader in the development of high quality, innovative educational programs and services. Keeping in mind that students come first, we will model best practice in instructional and service delivery, student access, community involvement, and accountability."

This guide aligns with both the district's mission and the stated goals of the ALAS program. It will be freely available to our students and will help them develop the skills they need for their academic careers. It can be emailed to the student as a document. It can be part of an online portion to the linear algebra course, inviting comment and discussion. Our instructors can take the guide and add to it. I envision it being a vibrant and growing document which engages both our students and our staff, under the aegis of ALAS.